CHAPTER SIX

ELECTROSTATICS

Electrostatics is the study of stationary or static charges and the forces between them. In this chapter, we will review Coulomb's law, which gives the electrostatic force between two charges and then discuss the concept of the electric field, which gives the electrostatic force per unit charge. The topic of electric potential energy is discussed along with the related topic of the electric potential, which is electric potential energy per unit charge. One can think of the electric potential being related to the electric potential energy in the same way as the electric field is related to the electric force. Associated with the electric field and electric potential are the concepts of field lines and equipotential lines which are reviewed separately. Finally, a detailed discussion of the electric dipole is presented with a worked example of a real life dipole, the H_2O molecule.

$V = QC$

$= Ed$

CHARGES

Charge may be either positive or negative. A positive charge and a negative charge attract one another; positive repels positive; and negative repels negative. To summarize: **unlike charges attract, like charges repel.** The force that exists between stationary charges is known as the **electrostatic** force.

Net charge can appear on a macroscopic object due to friction. If a glass rod is rubbed on a piece of silk, electrons, which are negatively charged, flow from the glass rod to the silk cloth. This results in the glass rod being positively charged and the silk cloth being negatively charged. The rod and cloth then attract each other; this is known as static cling.

The SI unit of charge is the **Coulomb** and the **fundamental unit of charge** is:

$$e = 1.60 \times 10^{-19}\ \text{C}$$

Both protons and electrons have this amount of charge, though protons are positively charged ($q = +e$), and electrons are negatively charged ($q = -e$).

Note:

Materials are usually electrically neutral, but can sometimes be charged by stripping off loose surface charges simply by rubbing.

COULOMB'S LAW

electric field

Coulomb's law gives the magnitude of the electrostatic force F between two charges q_1 and q_2 whose centers are separated by a distance r:

$$F = K\frac{q_1 q_2}{r^2}$$

Note:

Coulomb's law gives the magnitude of the force of q_1 on q_2 = the force of q_2 on q_1. This is analogous to Newton's law of gravity which gives force of m_1 on m_2 = force of m_2 on m_1.

Note:

The direction of the force between two charges is determined by the signs of the charges, i.e., like charges repel, unlike charges attract.

MCAT Favorite:

When the distance between two charges is doubled, the force is reduced to 1/4 of the original force, i.e., $F \propto 1/r^2$.

where k is called **Coulomb's constant** or the **electrostatic constant,** and is a number that depends on the units used in the equation. In SI units $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9\ N{\bullet}m^2/C^2$, where $\varepsilon_0 = 8.85 \times 10^{-12} C^2/N{\bullet}m^2$ and is called the **permittivity of free space.**

Coulomb's law in SI units is therefore:

$$F = K\frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2} = 8.99 \times 10^9\frac{q_1q_2}{r^2}$$

where the force F is in Newtons, the charges q_1 and q_2 are in Coulombs, and the distance r is in meters. The direction of the force may be obtained by remembering that unlike charges attract and like charges repel. The force always points along the line connecting the centers of the two charges.

Example: A positive charge is attracted to a negative charge a certain distance away. The charges are then moved so that they are separated by twice the distance. How has the force of attraction changed between them?

Solution: Coulomb's law states that the force between two charges varies as the inverse of the square of the distance between them. Therefore, if the distance is doubled, the square of the distance is quadrupled and the force is reduced to 1/4 of what it was originally. Note that it was not necessary to know the distance or the units being used, but only the fact that the distance was doubled and that the relation was an inverse square law.

Example: Negatively charged electrons are electrostatically attracted to positively charged protons (together they form hydrogen atoms). Because electrons and protons have mass, they will be gravitationally attracted to each other as well. Compare the two forces using Coulomb's law and Newton's law of gravitation. (Use $m_p = 1.67 \times 10^{-27}$ kg, $m_e = 9.11 \times 10^{-31}$ kg, and a Bohr radius separation between the electron and proton so that $r = 5.29 \times 10^{-11}$m.)

Solution: Both Coulomb's law and Newton's law state that the attractive forces between the electron and proton vary as the inverse of the square of the distance between them. As calculated in Chapter 2, the gravitational attractive force is:

$$F_N = \frac{Gm_pm_e}{r^2}$$

$$= \frac{(6.67 \times 10^{-11})(1.67 \times 10^{-27})(9.11 \times 10^{-31})}{(5.29 \times 10^{-11})^2}$$

$$= 3.63 \times 10^{-47}\ N \approx 10^{-47}\ N$$

On the other hand, the magnitude of the electrostatic attractive force is:

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{q_pq_e}{r^2}$$

$$= \frac{(8.99 \times 10^{9})(1.60 \times 10^{-19})(1.60 \times 10^{-19})}{(5.29 \times 10^{-11})^{2}}$$

$$= 8.22 \times 10^{-8}\ \text{N} \approx 10^{-7}\ \text{N}$$

Note that the electrostatic attraction between the electron and proton is stronger than the gravitational attraction by a factor of approximately 10^{40}.

ELECTRIC FIELD

Every electric charge sets up a surrounding **electric field.** The electric field can be detected by the force it exerts on other electric charges. It is defined as the force on a stationary positive test charge q_0 divided by the charge. It is therefore a vector quantity given by:

$$E = \frac{F}{q_0}$$

why divide by q_0 → force/ quantity of charge ← is a convention

The electric field **E** points in the direction of the force **F** on the positive test charge q_0. In SI units, **E** is measured in Newtons/Coulomb, which equals Volts/meter. The Volt will be defined in the next section where the electric potential is discussed.

Given an electric field **E** in some region of space, any charge q_0 placed in the field experiences a force **F** given by:

$$F = q_0E$$

click on each variable & see a picture

In this vector equation, we keep the sign of the charge, so that the force **F** is in the direction of q_0**E**: that is, in the same direction as **E** itself if q_0 is positive, but in the opposite direction to **E** if q_0 is negative.

The force on a positive test charge q_0 placed a distance r from a charge q is given by Coulomb's Law:

$$F = k\frac{qq_0}{r^2}$$

Using this equation and the fact that the electric field $E = F/q_0$, we get an equation for the electric field at any distance r from a charge q:

$$E = k\frac{q}{r^2}$$

The **direction** of the electric field vector is such that it points away from q if q is a positive charge, but it points towards q if q is a negative charge.

confusing!

In order to visualize the direction and magnitude of the electric field vector over a wide number of points, it is helpful to think of **field lines.** Field lines, or **lines of force**, as they are sometimes called, are imaginary lines that represent how a positive test charge would be accelerated in the electric field. For example, the field lines for a negatively charged particle such as an electron would point radially toward the charge, since the positive test charge would be attracted toward a negative charge. Similarly, the field lines point radially away from a positive charge, such as a proton, since the positive test charge would be repelled away from another positive charge.

Note:

The Electric field is a vector with units given by $E = \frac{F}{q_0}$ = Newtons/Coulomb.

Note:

A positive charge in an electric field feels a force in the direction of the field. A negative charge in an electric field feels a force in the direction opposite the field.

+ effect

Note:

The electric field of a positive charge points radially outward from the charge. The electric field of a negative charge points radially inward towards the charge.

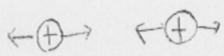
? ⇌ +
q_0
positive test charge

Note:

The electric field strength is stronger where the field lines are closer together and weaker where the field lines are farther apart.

Note:

Electric fields of separate charges add as vectors.



The direction of the electric field, at a given point, is always tangent to the field line at that point and in the same direction. Field lines also indicate the relative strength of the electric field. Where the field lines are closer together the electric field is stronger; where the field lines are farther apart the electric field is weaker.

For a collection of charges, the total electric field at a point in space is the **vector sum** of the electric field due to each charge:

$$\mathbf{E}_{total} = \mathbf{E}_{q_1} + \mathbf{E}_{q_2} + \mathbf{E}_{q_3} + \ldots \text{ (vector sum)}$$

The vector sum must be carried out using the rules of vector addition, as shown in the following example.

Example: A positive charge of $+1 \times 10^{-5}$ C is located one meter away from another positive charge of $+2 \times 10^{-5}$ C. At what point along the line between the two charges is the electric field equal to zero?

Solution: In order for the sum of two vectors to be zero, they must be equal in magnitude and opposite in direction. Because both of the charges are positive, the electric field vector of each charge points away from the charge. Along the line between the two charges the two electric field vectors point in opposite directions. If the charges were equal in magnitude the point at which the two fields have the same magnitude (and therefore where the resultant field is zero) would be exactly halfway between them. However, the charges are not equal, since one charge is half the charge of the other. Let x be the distance from the $+1 \times 10^{-5}$ C charge. The distance from the other charge is the total distance of one meter minus x, or $(1 - x)$.

Setting the magnitudes of the two fields equal to each other to find the distance x that will make them equal, we have:

$$k\frac{(1 \times 10^{-5})}{x^2} = k\frac{(2 \times 10^{-5})}{(1 - x)^2}$$

$$\frac{1}{x^2} = \frac{2}{(1 - x)^2}$$

$$2x^2 = (1 - x)^2$$

$$\sqrt{2}x = 1 - x$$

$$x(1 + \sqrt{2}) = 1$$

; too complex

$$x = \frac{1}{\sqrt{2} + 1}$$

$$x = 0.41 \text{ m}$$

As might be expected, this point is closer to the smaller charge since the field of the larger charge is stronger.

ELECTRIC POTENTIAL

Just as work is required to lift an object against the Earth's gravitational field, work must be done to move an electric charge in an electric field. The **electric potential** at a point is defined as the amount of work needed to move a positive test charge q_0 from infinity to that point divided by the test charge q_0:

$$* \quad V = \frac{W}{q_0} = U$$

hard to understand; want visual instead

In SI units electric potential is measured in **Volts** (V) where 1 Volt = 1 Joule/Coulomb. The electric potential at a distance r from a point charge q is:

$$* \quad V = k\frac{q}{r}$$

V is a scalar quantity whose sign is determined by the sign of the charge q. For a positive charge V is positive, but for a negative charge V is negative. For a collection of charges, the total electric potential at a point in space is the **scalar sum** of the electric potential due to each charge:

$$* \quad V_{total} = V_{q_1} + V_{q_2} + V_{q_3} + \ldots \text{ (scalar sum)}$$

Potential difference (voltage) is the difference in potential between two points. If V_a and V_b are the electric potentials at points a and b, then the potential difference between a and b is $V_b - V_a$. From the definition of electric potential, it follows that the potential difference between a and b can be expressed as:

$$* \quad V_b - V_a = \frac{W_{ab}}{q_0}$$

pos. chrg. so more V.

where W_{ab} is the work needed to move a test charge q_0 through an electric field from a to b. The work depends only on the potentials at the two points a and b, and is independent of the path. This means that like the gravitational force in Chapter 3, the electrostatic force is a conservative force.

Typical voltages encountered in medical research range from the millivolt (e.g., the 70 to 90 millivolt potential across a cell membrane), to 10 volts (the approximate pulse voltage of a pacemaker), to the tens of billions of volts (gigavolts or GV) used to accelerate protons for nuclear medicine purposes (as in the preparation of radioisotopes).

Note:

The electric potential, V, has units of Volts and is electric potential energy per unit charge, so Volt = Joule /Coulomb.

Note:

The electric potential of a positive/negative charge is a positive/negative number.

Note:

The electric potentials of separate charges add as scalars (numbers).

EQUIPOTENTIAL LINES

An **equipotential line** is one for which the potential at every point is the same. The potential difference between any two points on an equipotential line is zero. From the above equation it follows that no work is done when moving a test charge q_0 from one point to another on an equipotential line. Work will be done in going from one line to another, but the **work depends only on the potential difference of the two lines and not on the path.**

Example: In figure 6.1, an electron goes from point a to point b in the vicinity of a very large positive charge. The electron could be made to follow any of the paths shown. Which path requires the least work to get the electron charge from a to b?

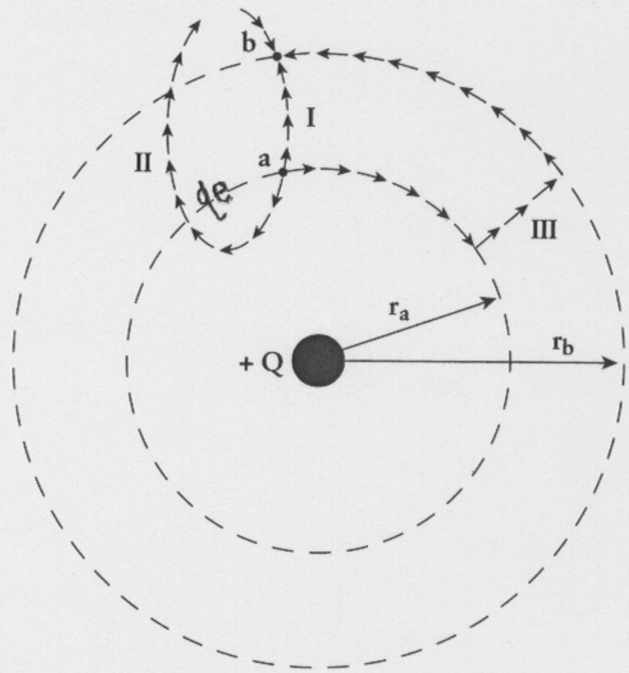


Figure 6.1

Solution: As stated, the **work depends only on the potential difference and not on the path,** so any of the paths shown would require the same amount of work in moving the electron from a to b, namely:

$$W_{ab} = q_e(V_b - V_a)$$
$$= q_e\left(k\frac{Q}{r_b} - k\frac{Q}{r_a}\right)$$

So paths I, II, and III all require the same amount of work to move the electron. (Note that W_{ab} is positive in this example since $r_a < r_b$ and $q_e = -e$).

MCAT Favorite:

The amount of work necessary to move a charge in an electric field depends only on the potential difference. Recall that in the case of gravity, the work required to move a mass depends only on the difference in height.

ELECTRIC POTENTIAL ENERGY

We have already defined the electric potential V at a point in space as the amount of work W required to move a positive test charge q_0 from infinity to that point divided by q_0. We now define the electric potential energy U of an arbitrary charge q at that point in space to be the amount of work needed to move it from infinity to the point. Using the definition of the electric potential we get:

$$U = W = qV$$

where V is the electric potential due to the other charges. Note that the sign of U depends on the signs of q and V. Since U = qV, it may be said that V = U/q; electric potential can also be thought of as electric potential energy per unit charge. When V is due to just one other charge Q, V is given by kQ/r, and U may be rewritten as:

Note:

Electric potential energy, U, equals charge times electric potential: $U = qV$. Change in electric potential energy equals charge times change in electric potential: $\Delta U = q\Delta V$.

; too complicated

$$U = k\frac{qQ}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$$

If the charges are both positive or both negative (in other words, like charges), U will be positive, but if one charge is positive and the other negative (that is, unlike charges), U will be negative.

Example: If a charge of +2e and a charge of −3e are separated by a distance of 3 nm, what is the potential energy of the system? (e is the fundamental unit of charge equal to 1.6×10^{-19} C.)

Solution:

$$U = k\frac{qQ}{r}$$

From the question stem we know that q = +2e, Q = −3e, and r = 3nm = 3×10^{-9}m. So, putting these numbers into the equation, and approximating k as 9.0×10^9:

$$U = (9 \times 10^9)\frac{(2)(1.6 \times 10^{-19})(-3)(1.6 \times 10^{-19})}{(3.0 \times 10^{-9})}$$

$$= -4.6 \times 10^{-19}\ \text{J}$$

Note:

Electric potential energy of a two charge system can be positive (two like charges) or negative (two unlike charges).

THE ELECTRIC DIPOLE

Two equal and opposite charges a small distance d away from each other form what is called an **electric dipole.** Suppose there is a dipole with charges +q and −q, as shown in figure 6.2.

In A Nutshell:

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a small distance.

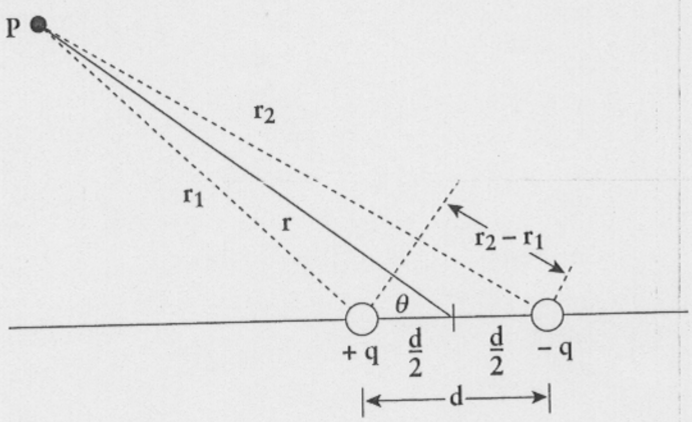


Figure 6.2

The potential at any point P is given by the sum of the two potentials:

$$V = k\frac{q}{r_1} - k\frac{q}{r_2}$$

$$= kq\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

For points relatively far from the dipole (compared to d), $r_1 r_2 \cong r^2$ and $r_2 - r_1 \cong d \cos\theta$. With these approximations the potential becomes:

$$V = k\frac{qd}{r^2}\cos\theta$$

The product of qd is defined as the **dipole moment p** with SI units of C•m. This is a vector quantity. Its magnitude is equal to the product qd, and its direction lies along the line connecting the charges (dipole axis) and points from the negative charge toward the positive charge. (Beware! Chemists often reverse this convention, having **p** point from the positive toward the negative charge.) In terms of dipole moment, one can rewrite the dipole potential as:

$$V = k\frac{p}{r^2}\cos\theta$$

Note that the potential is zero for $\theta = 90°$ and that this is the plane that lies halfway between +q and –q (called the perpendicular bisector of the dipole).

The electric field produced by the dipole at any point is the vector sum of each of the individual fields due to each of the two charges. Along the perpendicular bisector of the dipole the magnitude of the electric field can be approximated as:

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

The field will point in the opposite direction to **p**.

Example: The H_2O molecule has a dipole moment of 1.85 D, where D = Debye unit = 3.34×10^{-30} C•m. Calculate the electric potential due to an H_2O molecule at a point 89 nm away along the axis of the dipole. (Use $k = 9 \times 10^9$ N•m²/C².)

Solution: Since the question asks for the potential along the axis of the dipole, the angle θ is given by 0°. Substituting the values into the equation for the dipole potential and multiplying 1.85 D by 3.34×10^{-30} to convert it to C•m:

$$V = k\frac{p}{r^2}\cos\theta$$

$$= (9 \times 10^9)\,\frac{(1.85)(3.34 \times 10^{-30})(\cos 0°)}{(89 \times 10^{-9})}$$

$$= 7 \times 10^{-13}\ \text{V}$$

too complex

Now consider the case when an electric dipole is placed in a uniform external electric field. If there is no field present, the dipole moment will assume any random orientation. With a uniform external electric field present, however, each of the equal but opposite charges that make up the dipole will feel a force exerted on

Note:

The net force on a dipole is the sum of the forces on the two charges. The net torque on the dipole is the sum of the torques due to the forces on the two charges.

it by the external electric field. The net force will be zero, since the force on each charge is equal in magnitude but opposite in direction. The dipole therefore feels no translational force. However, there will be a nonzero torque about the center:

$$\begin{aligned}\tau &= F\frac{d}{2}\sin\theta + F\frac{d}{2}\sin\theta \\ &= Fd\sin\theta \\ &= qEd\sin\theta \\ &= (qd)E\sin\theta \\ &= pE\sin\theta\end{aligned}$$

where p is the magnitude of the dipole moment (p = qd), E is the magnitude of the uniform external electric field, and θ is the angle the dipole moment makes with the electric field. This torque will cause the dipole to reorient itself by rotating, so that its dipole moment, **p**, aligns with the electric field **E.** This is shown in figure 6.3.

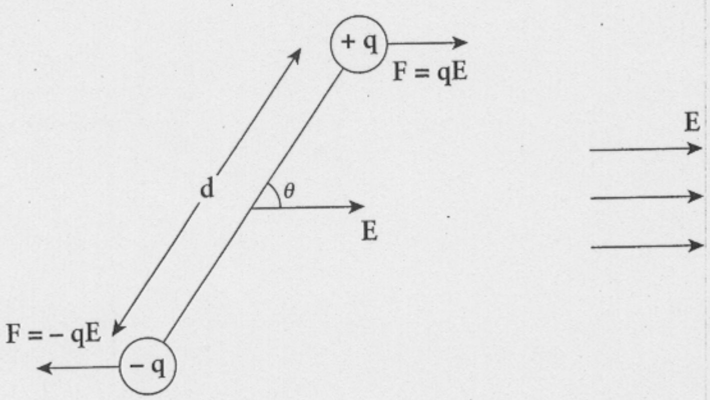
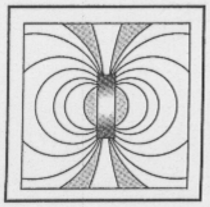


Figure 6.3

For more information on dipole moments see Chapter 3 of the General Chemistry Notes.



CHAPTER SEVEN

MAGNETISM

6,7,8
※ Try to teach all of these
E & M chapters combined
Layer concepts

In this chapter we will review the subject of magnetism. Unlike electrostatics where electric charges create electric fields which exert forces on other electric charges, magnetism has no fundamental magnetic charges. Instead, magnetic fields are created by moving charges, currents in wires, and permanent magnets. These magnetic fields, in turn, exert magnetic forces on the very things that create them, i.e., moving charges, currents in wires, and permanent magnets.

The first half of the chapter is concerned with the determination of the magnetic force due to a given magnetic field. Since force is a vector, both the magnitude and direction of the magnetic force are considered. The second half of the chapter then examines sources of magnetic fields, including a brief review of magnetic materials, and also a discussion of the two most common current configurations: the straight wire and the loop of wire.

In A Nutshell:

Magnetic fields are created by moving charges and permanent magnets, and in turn exert forces on moving charges and permanent magnets.

THE MAGNETIC FIELD

In discussing the magnetic force on moving charges and on current carrying wires, we will assume the presence of a fixed and uniform magnetic field **B**. Of course, this field must be produced by some external source such as a magnet or arrangement of current carrying wires, but for our purposes we are only concerned with the strength and direction of this field.

Like all physical quantities, magnetic fields have units. The SI unit of the magnetic field is the Tesla (T) where $1\text{ T} = 1\,\frac{\text{N}\cdot\text{s}}{\text{m}\cdot\text{C}}$ Small magnetic fields are sometimes measured in Gauss where 1 Tesla = 10^4 Gauss.

A. magnetic FORCE ON A MOVING CHARGE

When a charge moves in a magnetic field, a magnetic force is exerted on it. This force, like all forces, is a vector. The **magnitude** of F is given by:

$$F = qvB \sin\theta$$

In using this formula, θ is the smallest angle between the vectors **qv** and **B** (more on qv below), q is the charge of the moving particle, v is the particle's speed, and $B = |\mathbf{B}|$ is the magnitude of the magnetic field vector.

Right-Hand Rule for the direction of the magnetic force on a moving charge. Turning our attention to the **direction** of the magnetic force, we should

MCAT Favorite:

When a charge moves parallel to (θ = 0°) or anti-parallel to (θ = 180°) a magnetic field, the magnetic force is zero.

first note that **qv** is a vector that depends on the velocity vector **v** and the sign of the charge q. If q is nonzero and positive (positive charge), then q**v** points in the same direction as **v.** If q is nonzero and negative (negative charge), then q**v** points in the opposite direction as **v.** (If q or **v** is zero, then the magnetic force will be zero.) The direction of the magnetic force will be **perpendicular** to the plane defined by q**v** and **B,** but this could be either of two directions. To find the correct direction, let the thumb of the **right hand** (left-handed people must be careful to use the correct hand) point in the direction of the vector q**v** (that is, parallel to **v** if q is positive and antiparallel to **v** if q is negative). Let the remaining fingers of the **right hand** point in the direction of **B.** Your **palm** now points in the direction of **F,** the magnetic force on q.

(Note: The right-hand rule as stated above may differ from what you have previously learned. A different version would have the right index finger in the direction of q**v** and right middle finger in the direction of **B** and, holding the thumb perpendicular to these two fingers, the right thumb points in the direction of **F.** It is important only to get the direction correct no matter which rule you use. **If you have committed to memory another version of the rule, and it works, then feel free to use it.)**

Because of the three-dimensional nature of problems involving magnetic fields, scientists have chosen the following conventions to denote magnetic fields going into the page, or coming out of the page. The symbol x represents a field going into the page. The x represents the tail end of an arrow travelling into the page. The symbol • represents a field coming out of the page. The • represents the tip of an arrow coming out of the page.

Note:

To find the direction of the magnetic force, use whichever (valid) right hand rule you prefer.

Note:

The symbol 'x' means the magnetic field direction is into the page, and the symbol '•' means the magnetic field direction is out of the page.

Example: Suppose a proton, whose charge is $+1.6 \times 10^{-19}$ C, is moving with a speed of 15 m/s in a direction parallel to a uniform magnetic field of 3.0 T. What is the magnitude and direction of the magnetic force on the proton?

Solution: Because the proton is positively charged, the vector q**v** is in the same direction as **v,** which is the same direction as **B** as stated in the problem. Since q**v** and **B** are pointing in the same direction, the angle between the vectors is zero. Since $\sin 0° = 0$ and $F = qvB \sin \theta$, the magnetic force on the proton is zero, too. Note that if the charge had been negative (an electron, for example), the angle between q**v** and **B** would have been 180° and since $\sin 180° = 0$, the magnetic force on a negative charge moving parallel to a uniform magnetic field would be zero as well. In general, the magnetic force on a moving charge will be zero if the charge is moving parallel or antiparallel to the magnetic field.

Example: Suppose a proton whose charge equals $+1.6 \times 10^{-19}$ C is moving with a speed of 15 m/s toward the top of the page and through a uniform magnetic field of 3.0 T directed into the page (see figure 7.1(a)). What is the magnitude and direction of the magnetic force on the proton?

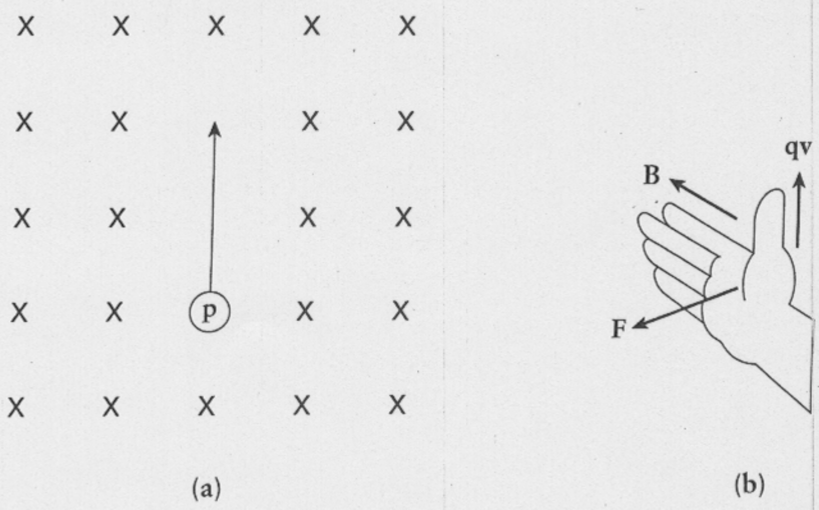


Figure 7.1

Solution: Because the proton is positively charged, the vector **qv** is in the same direction as **v**, which is perpendicular to **B** as stated in the problem. (**B** is perpendicular to the plane of the page.) Because qv and **B** are perpendicular, the angle between the vectors is $\theta = 90°$, and since $\sin 90° = 1$, the magnetic force on the proton is:

$$\begin{aligned} F &= qvB \sin \theta \\ &= qvB \\ &= (1.6 \times 10^{-19})(15)(3.0) \\ &= 7.2 \times 10^{-18} \text{ N} \end{aligned}$$

By holding the thumb of the **right hand** so that it is directed toward the top of the page, then holding the remaining fingers of the **right hand** so that they point towards (into) the page, one's **right hand** palm points to the left (see figure 7.1(b)). Hence, the proton is deflected to the left on its upward journey. As the velocity of the proton changes, so does the magnetic force that it experiences. Note that if the charge had been negative (an electron, for example), the angle between **qv** and **B** still would have been 90°, but the right-hand rule would have required that **qv** points toward the bottom of the page, meaning one's right-hand palm would point to the right. Hence an electron is deflected to the right on its upward journey. One can readily see that the direction of the magnetic force on a negative charge moving through a magnetic field is opposite to the direction of the magnetic force acting on a positive charge moving in the same direction.

When a charged particle moves **perpendicular** to a **constant, uniform magnetic field,** the resulting motion is circular motion with constant speed in the plane perpendicular to the magnetic field. A centripetal force is always associated

Note:

The magnetic force on a moving charged particle is always perpendicular to both the velocity of the charge and the magnetic field direction. Thus, a magnetic force does no work. (See Chapter 3.)

*neg: reverse

Note:

The direction of the magnetic force on a negative charge is opposite to the direction of the force on a positive charge moving in the same direction as the negative one.

with circular motion. In this case the centripetal force is the magnetic force ($F = qvB$). Since the centripetal force equals mv^2/r, we get:

$$F = qvB = \frac{mv^2}{r}$$

From this equation one can solve for the orbit radius, the magnetic field, and so on:

$$r = \frac{mv}{qB} \qquad B = \frac{mv}{qr}$$

Example: Suppose the proton of the previous example is allowed to circle (counterclockwise) in the same perpendicular magnetic field of 3.0 T with the same speed of 15 m/s (as in figure 7.2(a)). What is the orbit radius r? (The mass of a proton is 1.67×10^{-27} kg.)

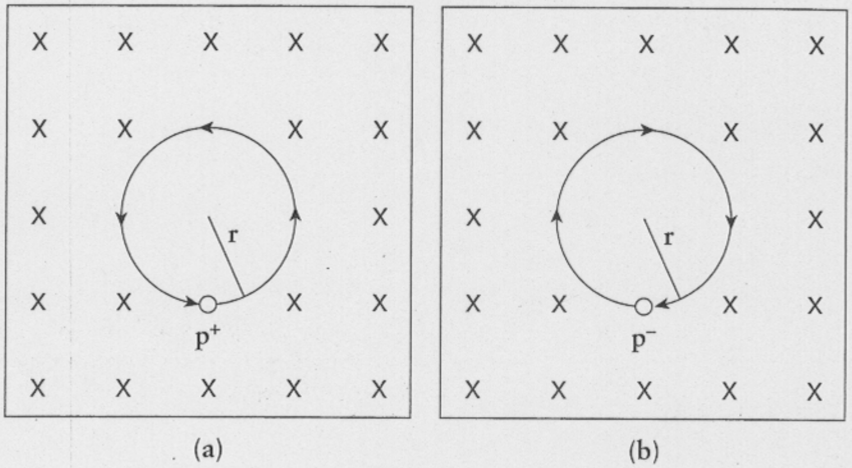


Figure 7.2

Solution: By equating the centripetal force to the magnetic force and solving for the orbit radius as shown above:

$$r = \frac{mv}{qB}$$

$$= \frac{(1.67 \times 10^{-27})(15)}{(1.6 \times 10^{-19})(3)}$$

$$= 5.2 \times 10^{-8} \text{ m}$$

Note that the direction of the magnetic force on a negative charge moving through a uniform magnetic field is opposite to the direction of the magnetic force acting on a positive charge moving in the same direction. Therefore, if the charge had been negative (an antiproton, for example, which has the mass of the proton but is negatively charged), it would have circled in the **clockwise** direction with the same orbit radius. (See figure 7.2 (b))

Note:

A charged particle moving perpendicular to a constant, uniform magnetic field undergoes uniform circular motion. The centripetal force in this case is the magnetic force on the charge.

Note:

To determine sense of rotation (clockwise or counterclockwise) of a charged particle undergoing uniform circular motion, apply right-hand rule for magnetic force on the particle.

Note:

For a given field direction, if positive charges circle clockwise, then negative charges circle counterclockwise.

B. CURRENT

Electric current will be discussed more completely in Chapter 8. However, it is important to realize that when two points at different electric potentials are connected with a conductor (such as a metal wire), charge flows between the two points. The flow of charge is called an **electric current.** The magnitude of the current i is the amount of charge Δq passing through the conductor per unit time Δt, or in the form of an equation:

$$i = \frac{\Delta q}{\Delta t}$$

The SI unit of current is the Ampere (1 A = 1 Coulomb/second).

Charge is transmitted by a flow of electrons in a conductor. Since electrons are negatively charged, they go from lower potentials to higher potentials. But, **by convention,** the direction of **current** is the direction in which **positive charge** would flow, or from high to low potential. **Thus the direction of current is opposite to the direction of electron flow.**

Note:

Current is in the direction that positive charges would flow.

magnetic

C. FORCE ON A CURRENT-CARRYING WIRE

Since moving charge is subject to magnetic forces and electric current is a flow of charge, it should come as no surprise that magnetic forces can act on a current-carrying wire. For a straight wire of length L carrying a current i in a direction that makes an angle θ with a uniform magnetic field **B**, the magnitude of the magnetic force on the current-carrying wire is:

$$F = iLB \sin \theta$$

Note:

A current in a magnetic field behaves similarly to a positive charge moving in a magnetic field since current is a collection of moving positive charges.

The direction of the force is given by a simple right-hand rule, the **right-hand rule for the magnetic force on currents.** The force will be **perpendicular** to the plane defined by **B** and the direction of the current flow, but this could be either of two directions. To find the correct direction let the thumb of the **right hand** (left-handed people must be careful to use the correct hand) point in the direction of the current i. Now let the remaining fingers of the **right hand** point in the direction of **B.** The palm of the **right hand** now points in the direction of **F,** the magnetic force on the current-carrying wire. (Note: This rule is virtually the same as the rule given above for moving charges. Again, you should feel free to use any right-hand rule that you have committed to memory and that gives the correct direction.)

Example: Suppose a wire of length 2.0 m is conducting a current of 5.0 A toward the top of the page and through a 30 Gauss uniform magnetic field directed into the page (see figure 7.3(a)). What is the magnitude and direction of the magnetic force on the wire?

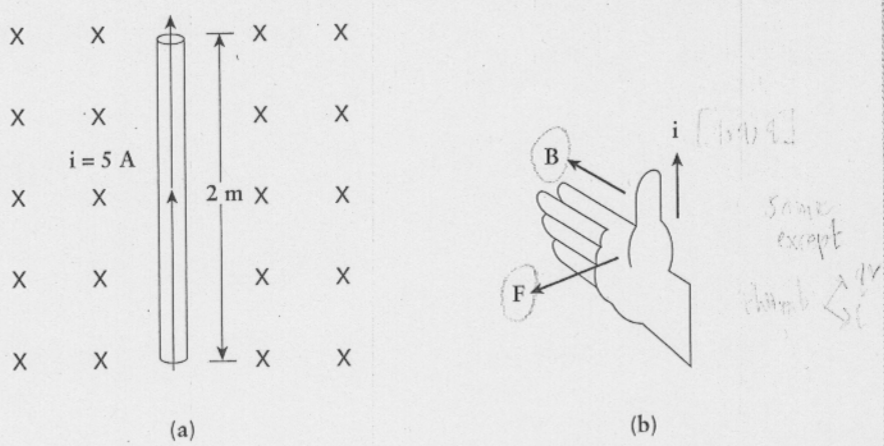


Figure 7.3

Note:

For direction of magnetic force on a current, use same right-hand rule as for charges, but replace velocity direction with current direction.

Solution: Since 1 T = 10^4 Gauss, 1 Gauss = 10^{-4} T, 30 Gauss = 30×10^{-4} T = 3×10^{-3} T. The wire is conducting current toward the top of the page and the magnetic field points into the page; therefore, the current is perpendicular to **B**. The angle between them is $\theta = 90°$, and since $\sin 90° = 1$, the magnetic force on the wire is:

$$\begin{aligned} F &= iLB \sin\theta = iLB \\ &= 5.0(2.0)(3.0 \times 10^{-3}) \\ &= 3.0 \times 10^{-2}\text{ N} = 0.03\text{ N} \end{aligned}$$

By holding the thumb of the **right hand** so that it is directed toward the top of the page, then holding the remaining fingers towards (into) the page, the palm of the **right hand** points to the left. Hence the force on the wire is to the left.

Note:

Wires are electrically neutral, so there's no magnetic force if the wire simply moves through a magnetic field with no current in the wire.

SOURCES OF MAGNETIC FIELD

The previous section dealt with the magnetic force on a moving charge and a current-carrying wire, but did not discuss how the field was generated. Any moving charge creates a magnetic field. Magnetic fields may be set up by the "flow" of charge in permanent magnets, or electric currents, or simply by individual moving charges (e.g., an electron moving through space). This section deals only with permanent magnets and current-carrying wires, but it is important to realize that each of these sources of magnetic field has, in one sense or another, a flow of charge or a current—it is the movement of charge that gives rise to the magnetic field.

As with electric fields, magnetic **field lines** can be used to visualize the magnetic field. At any point along a field line the magnetic field itself is in the tangential direction.

Flashback To Chapter 6:

Magnetic field lines are analogous to electric field lines, i.e., the magnetic field is tangent to the magnetic field line at any point.

A. MAGNETIC MATERIALS

Materials are classified as diamagnetic, paramagnetic, and ferromagnetic. In a **diamagnetic material**, the individual atoms have no net magnetic field. Diamagnetic materials will be repelled from the pole of a strong bar magnet, so they are sometimes called weakly antimagnetic. In **paramagnetic** and **ferromagnetic** materials, the individual atoms do have a net magnetic field, but normally these individual atomic fields are randomly oriented so the material itself exhibits no net magnetic field. In a paramagnetic material under certain conditions, some degree of alignment of the individual atomic magnetic fields can occur. Paramagnetic materials will be attracted towards the pole of a strong bar magnet, so they are sometimes called weakly magnetic. In a ferromagnetic material a special effect takes place when the temperature drops below a critical value that allows a high degree of alignment of the magnetic fields of the individual atoms to occur. Above this critical temperature, called the Curie temperature, the material is paramagnetic. Ferromagnetic materials are sometimes called strongly magnetic and include iron, nickel, and cobalt. When the Curie temperature is above room temperature, ferromagnetic materials are permanently magnetized at room temperature (for example, the familiar bar magnet).

When a paper with iron filings is placed on top of a permanent bar magnet, the iron filings tend to form lines connecting the top of the magnet to the bottom of the magnet. The iron filings are showing the **magnetic field lines.** All bar magnets have a **north** and **south** pole. The north pole is the place where the magnetic field lines emerge; the south pole is where they enter. Given two bar magnets, opposite poles attract each other, like poles repel.

Real World Analogy:

A compass is simply a small permanent magnet designed so that it can rotate to whatever direction it naturally seeks. In particular, the compass needle will align itself with the local magnetic field direction in such a way that the north pole of the compass magnet points in the field direction. The earth also acts like a permanent magnet, having north and south magnetic poles and field lines that go from the north to the south magnetic pole. So when the north pole of a compass magnet points along the local field line of earth's magnetic field it is actually pointing towards a magnetic south pole. Thus, what we call the northern direction is actually in the direction of a magnetic south pole.

B. CURRENT-CARRYING WIRES

A current-carrying wire will produce a magnetic field in its vicinity. The magnetic field of a current carrying wire is the vector sum of the magnetic fields due to the individual moving charges that comprise the current. The final result depends on the shape of the wire. Special cases include a **long straight wire** and the **center of a circular loop of wire.**

At a perpendicular distance r from an infinitely long and straight current-carrying wire the magnitude of the magnetic field produced by the current i in the wire is given by:

B

eq. sub

$$B = \frac{\mu_0 i}{2\pi r}$$

(m)

where μ_0 is the **permeability of free space** = $4\pi \times 10^{-7}$ Tesla • meter/Ampere = 1.26×10^{-6} T•m/A. The above equation shows that for a long straight wire, the field strength drops off with distance.

The magnetic field lines are concentric perpendicular circles about the wire. You can use a **right-hand rule to find the direction of the magnetic field produced by a long straight wire.** This rule differs from the previous ones. In this rule your **right thumb** points in the direction of the current. Your remaining **right fingers** mimic the circular magnetic field lines and curl around the wire. Your fingers now show you the direction of the magnetic field lines and the direction of B itself at any point. Note that this rule differs from the previous two in that it gives the direction of the field lines produced by the current instead of starting with a given direction of **B** to find the direction of a force. Also note that, as shown in a later example, this rule may be applied to current loops as well as straight wires.

Example: A straight wire carries a current of 5 A toward the top of the page *(see figure 7.4(a)). What is the magnitude and direction of the* magnetic field at point P, which is 10 cm to the left of the wire? What is the magnitude and direction of the magnetic field at point Q, which is 2 cm to the right of the wire?

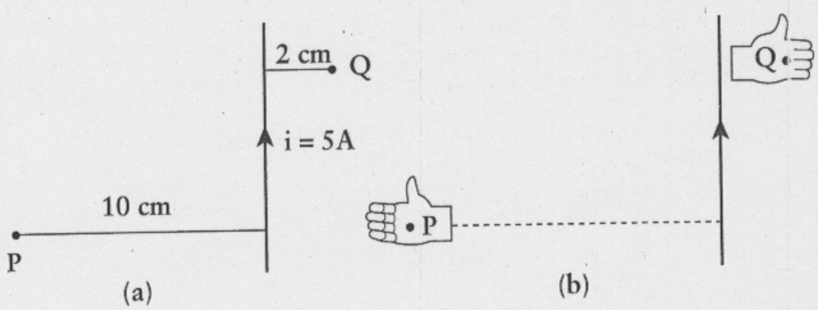


Figure 7.4

Note:

Magnetic field lines come out of north poles and go into south poles. Opposite magnetic poles attract and like poles repel.

easy
2-step

Note:

Magnetic field lines encircle currents. Use the right-hand rule (different from the rule for magnetic force on a moving charge) to find the field direction.

Note:

For a wire in the plane of the page, the field lines will go into the page on one side of the wire and come out of the page on the other side of the wire.

Solution: To find the magnitude at point P:

$$B = \frac{\mu_0 i}{2\pi r}$$

$$= \frac{(4\pi \times 10^{-7})(5)}{2\pi(0.1)}$$

$$= 10^{-5}\ \text{T} = 0.1\ \text{Gauss}$$

To find the magnitude at point Q:

$$B = \frac{(4\pi \times 10^{-7})(5)}{2\pi(0.02)}$$

$$= 5 \times 10^{-5}\ \text{T} = 0.5\ \text{Gauss}$$

Now to get the direction of the field for each of these points, we use the Right-Hand Rule. Hold your **right thumb** towards the top of the page. Now curl your fingers around the wire. At Q your fingers should point into the page. Keep curling around and you notice that at point P your fingers come out of the page. (See figure 7.4 (b)) So your answer should be: **B** (at P) = 0.1 Gauss, pointing out of the page, and **B** (at Q) = 0.5 Gauss, pointing into the page. Note that as we move farther from the wire, the magnitude of magnetic field decreases.

Note:

For a circular loop of wire, the equation $\frac{\mu_0 i}{2r}$ *only gives the magnetic field at one point in space, the point at the center of the loop.*

The magnitude of the magnetic field at the center of a circular loop of current-carrying wire of radius r is:

$$B = \frac{\mu_0 i}{2r}$$

Notice that these two laws for magnetic fields look similar. For the long straight wire, r refers to the perpendicular distance from the wire and gives B for any point away from the wire. However, r in the second case is the radius of the loop and the expression gives the magnetic field at the loop's center point only. The following example illustrates how to find directions.

Example: Suppose a wire is formed into a loop that carries current clockwise (that is, electrons flow counterclockwise) as in figure 7.5(a). Find the direction of the magnetic field produced by this loop:

a. within the loop.
b. outside of the loop.

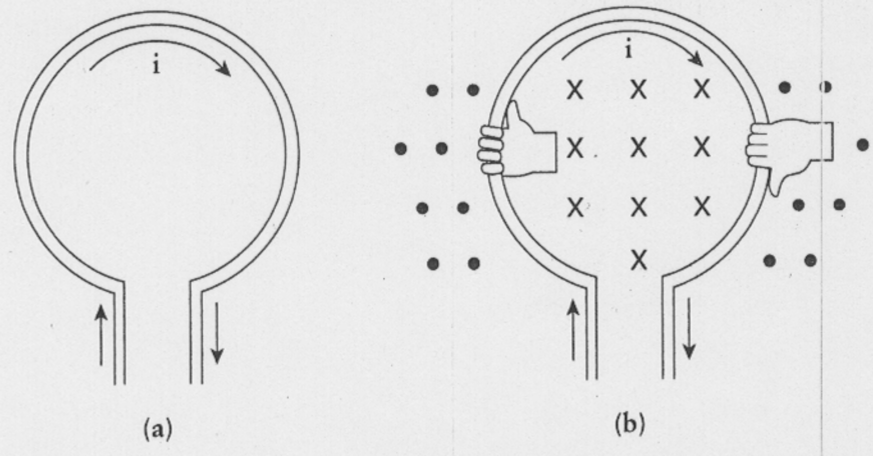


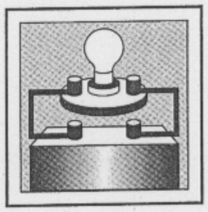
Figure 7.5

Solution: Look at figure 7.5(b). By holding your **right thumb** anywhere around the loop in the direction of current flow (clockwise) and encircling the wire with the remaining fingers of the **right hand,** your **right fingers** should point:

a. into the page. Thus the magnetic field within the loop points into the page.
b. out of the page. Thus the magnetic field outside the loop points out of the page.

Note:

Since the magnetic field circles around a current, it will be into the page on one side of a current loop (inside or outside) and out of the page on the other side.



CHAPTER EIGHT

DC AND AC CIRCUITS

Electric circuits pervade our everyday world, existing in myriad forms in the various necessities of modern day living, most notably, TV's, VCR's, and stereos. In this chapter we will review the essentials of DC circuits, touching only briefly and qualitatively on the subject of AC circuits. Included are the usual topics of DC circuit theory: emf, resistance, power dissipated by resistors, Kirchhoff's laws, parallel and series resistor circuits, capacitors, parallel and series capacitor circuits, and a brief discussion of dielectrics. Although the topic of DC circuits can be a place to encounter a substantial amount of algebra when solving complicated circuits, the emphasis on the MCAT and in this chapter is on the essential concepts involved and on applying those concepts in simple situations. Let's begin with a short review of conductors and insulators, the essential materials of the wires of any circuit.

Some materials allow electric charge to move freely within the material. These materials are called electrical **conductors.** Metal atoms can easily lose one or more of their outer electrons, which are then free to move around in the metal. This makes most metals good electrical conductors. In most conductors, the positive ions remain fixed and the liberated electrons are free to move.

In other materials electric charge is bound to the constituent atoms and is not free to move. These materials severely retard the flow of electricity and are called **insulators.** Most nonmetals are good insulators.

The wires to most appliances have a conducting core of copper wire perhaps, with an insulating sheath of some plastic. The copper wire conducts the electricity to the appliance from the wall socket. The insulating sheath protects you from touching the wire and getting an electric shock.

DIRECT CURRENT

A. CURRENT AND CIRCUIT VOLTAGE

The flow of charge is called an **electric current.** The magnitude of the current i is the amount of charge Δq passing a given point per unit time Δt, and is given by:

$$i = \frac{\Delta q}{\Delta t}$$

↑ Δ volume / second
Δ make go faster

The SI unit of current is the **Ampere** (1 A = 1 Coulomb/second). The two basic types of current flow are **direct current** (DC), where the charge flows in one direction only, and **alternating current** (AC), where the flow changes direction periodically. AC current will be discussed later.

Note:

Current flows from higher potential (positive terminal), to lower potential (negative terminal), analogous to masses which naturally fall (flow) from higher potential energy to lower potential energy.

When two points at different electric potentials are connected by a conductor (such as a metal wire), charge flows between the two points. In a conductor, only negatively charged electrons are free to move. These act as the charge carriers, and move from low to high potentials. By convention, however, the direction of the **current** is taken as the direction in which **positive charge** would flow, from high to low potential. **Thus the direction of current is opposite to the direction of electron flow.**

A voltage (potential difference) can be produced by an electric generator, a voltaic cell, or by a group of cells wired into a battery. **Electromotive force** (emf or ε) is the name given to the voltage across the terminals of a cell when no current is flowing. Electromotive force should not be confused with a force or an electric field; it is a potential difference and is measured in Volts.

Note:

Emfs are potential differences that cause current to flow. They are not forces!

Because cells typically have a small internal resistance r_{int} of their own, the voltage they actually furnish to a circuit is reduced by ir_{int}, where i is the current supplied by the cell. The voltage V across the terminals of the cell when current is flowing out, is given in terms of the cell's emf and internal resistance by:

$$V = \varepsilon - ir_{int}$$

Note that if the cell is supplying no current ($i = 0$), or if the cell has no internal resistance ($r_{int} = 0$), then $V = \varepsilon$. For cases in which the current supplied is greater than zero and the internal resistance is not negligible, then $V < \varepsilon$. When a cell is supplying current (discharging), the current flows out of the positive terminal and into the negative terminal. When a cell is being recharged, current from another source is sent into the positive terminal.

B. RESISTANCE

1. Resistance and Ohm's Law

Resistance R can be thought of as the opposition within a conductor to the flow of an electric current. This opposition takes the form of an energy loss or drop in potential. **Ohm's law** states that the voltage drop across a resistor is proportional to the current it carries, with R being the proportionality constant:

$$V = iR$$

collision w/ nuclei

VS. text

This equation applies to a single resistor within a circuit, to any part of a circuit, or to an entire circuit (provided one knows how to add resistances in series and parallel). Note that the current is unchanged as it passes through the resistor. This is because no charge is lost inside the resistor. Therefore, the current that is supplied to several resistors wired in series must all flow through each resistor. The SI derived unit of electrical resistance is the **Ohm** (Ω).

2. Resistance of a Conductor

The resistance of an object depends on its size, the type of material from which it is made, and its temperature. Specifically the resistance depends on:

In A Nutshell:

- *The resistance of a wire increases with increased length.*
- *The resistance of a wire decreases with increased cross-sectional area.*

more collision

a. Length (L)

Resistance is directly proportional to length. A longer conductor means greater resistance, because there is a longer path that current-carrying electrons must travel. For example, two wires, identical in every respect except that one is twice as long as the other, will have different resistances. The longer one will have twice the resistance of the shorter one.

b. Cross-sectional area (A)

The resistance of a conductor is inversely proportional to its cross-sectional area. An increase in cross-sectional area causes a decrease in resistance. This is because there is an increase in the number of conduction paths electrons can follow. For example, two wires, identical in every respect

respect except that one has twice the cross-sectional area of the other, will have different resistances. The thinner wire will have twice the resistance of the thicker wire.

c. **Resistivity of the conductor** (ρ)

Some materials are intrinsically better conductors of electricity than others. For example, copper conducts electricity much better than does glass. The number that characterizes the intrinsic resistance to current flow in a material is called the **resistivity** (ρ), where the SI unit of resistivity is the Ohm•meter. The resistivity is therefore defined as the proportionality constant relating a conductor's resistance to the ratio of its length over its cross-sectional area:

$$R = \rho \frac{L}{A}$$

can click on eq. and get pictures of variables substituted

d. **Temperature**

Most conductors have greater resistance at higher temperatures. This is due to increased thermal oscillations of atoms in the conductor which produce a greater resistance to electron flow. The resistivity can then be thought of as a function of temperature. A few materials, such as glass, pure silicon, and most semiconductors are exceptions to this general rule.

Note:

Resistivity, ρ, is a measure of the intrinsic resistance of a type of material, independent of length and cross-sectional area.

Real World Analogy:

Major home appliances such as ovens, washers, and dryers, require large currents for their operation. These large currents are achieved, in part, by using very thick wire. Thicker wire means a larger cross-sectional area which results in a smaller resistance and thus a larger current for a given voltage.

3. Power Dissipated By a Resistor

Electric potential is electric potential energy per unit positive charge. Since current is a flow of charge, it should come as no surprise that through a current-carrying resistor there is a **flow of energy.** In a resistor, this electric energy is converted into heat. The **rate** at which the energy loss occurs is equal to the power dissipated by the resistor and is given by:

$$P = iV$$

$(= I^2R)$

where i is the current flowing through the resistor and V is the potential drop across the resistor. Using Ohm's law this expression can be rewritten as:

$$P = i^2R = V^2/R$$

Note:

$P = iV$ means P = (charge/second) × (energy/charge). So P = energy/second, as power always does.

C. CIRCUIT LAWS

An electric circuit is a conducting path that usually has one or more voltage sources (such as a cell) connected to one or more **passive circuit elements** (such as resistors). This subsection deals primarily with voltages, resistances, and currents in DC circuits.

1. Kirchhoff's Laws

a. **At any point or junction in a circuit the sum of currents directed into that point equals the sum of currents directed away from that point.** This is a consequence of the **conservation of electric charge.**

Example: Three wires (a, b, and c) meet at a junction point P as in figure 8.1. A current of 5 A flows into P along wire a, and a current of 3 A flows away from P along wire b. What is the magnitude and direction of the current along wire c?

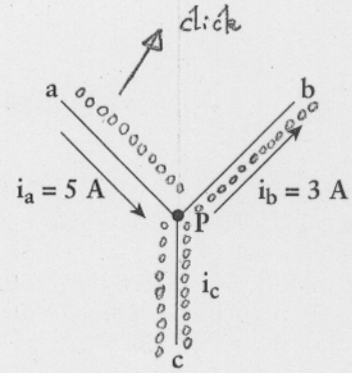


Figure 8.1

Solution: The sum of currents entering P must equal the sum of the currents leaving P. Assume for now that i_c flows out of P. If we find that it is negative, then we know that it flows into P.

$$i_a = i_b + i_c$$
$$i_c = 5 - 3$$
$$i_c = 2 \text{ A}$$

Thus a current of 2 A flows **out** of P along wire c. Note that the total current into and out of P is then zero.

b. **The sum of voltage sources is equal to the sum of voltage (potential) drops around a closed circuit loop.** This is a consequence of the conservation of energy: All the electrical energy supplied by a source gets fully used up by the rest of the circuit. No excess energy appears or disappears. (But remember that voltage is energy per unit charge not just energy.)

Note:

Energy is conserved in one complete loop of a circuit. Energy lost in the resistors is gained back in the battery.

Note:

Each additional resistor in a series of resistors increases the total resistance and thus decreases the total current.

2. **Resistors in Series**

It has already been mentioned that the same current flows through all the resistors in series and from the above laws we can deduce that voltage drops add in series. Therefore, using Ohm's law we find that resistances add in series (see figure 8.2). That is:

$$R_s = R_1 + R_2 + R_3 + \ldots + R_n$$

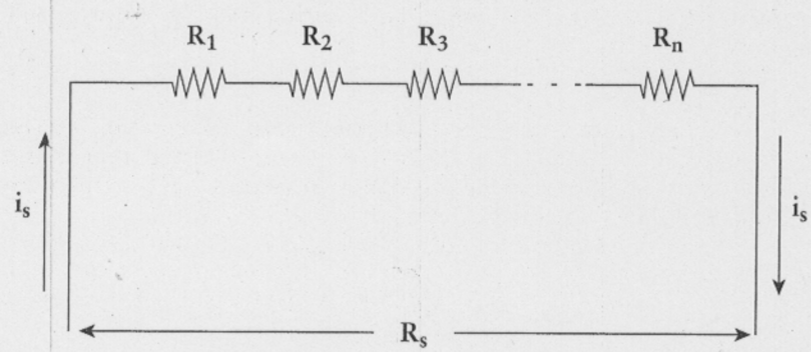
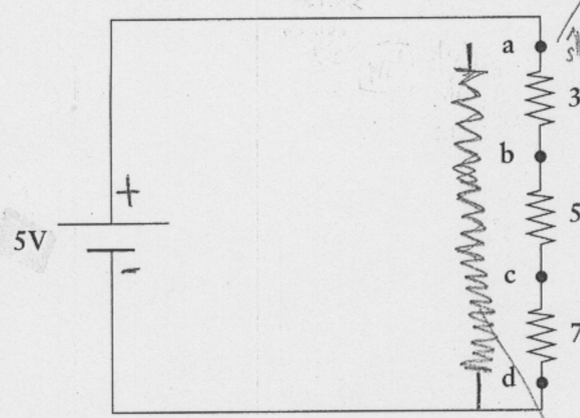


Figure 8.2

3

Example: A circuit is wired with one cell supplying 5 V (neglect the internal resistance of the cell) in series together with three resistors of 3 Ω, 5 Ω, and 7 Ω also wired in series as shown in figure 8.3. What is the resulting voltage across, and current through, each resistor of this circuit, as well as the entire circuit?



check w/ John

slow

i same @ each

V not the same

Gold:

~~Lenih~~

400

same number of currents

Voltage drop

Figure 8.3

12 x 15 = 180

$V_{drop} = I * R$

do electrons move @ constant speed? or slow down at resistors

Q.

Solution: The total resistance of the resistors is: 12

180 = drop / 15 = drop / electron

220

$$R_s = R_1 + R_2 + R_3$$
$$= 3 + 5 + 7$$
$$= 15\ \Omega$$

speed (V) concent. of electrons

Slider ↑ resistance see how Volt. affected

problem integrated into the flash animation itself

test question

Note:

Ohm's law can be applied to the entire circuit at once, using total resistance.

visualizing

energy dissipated

Now use Ohm's law to get the current through the entire circuit (since everything is in series, this is also the current through each element):

↑ field = ↑ current

Q. how to visualize?

↑ resistance ↓ current

$$i_s = \frac{V_s}{R_s} = \frac{5}{15} = \frac{1}{3}\text{A}$$

$\frac{5}{3}$

Now use Ohm's law for each of the resistors in turn. From a to b the voltage drop across R_1 is:

electrons:

V = 5V

V = 4.5V

V = 4V

C/S

$$iR_1 = \frac{1}{3}(3)$$
$$= 1.0\text{V}$$

across all electrons

point in time

Voltage: collision = less energy → potential drop

Note:

Ohm's law can be applied to each resistor separately.

Q. how calc. using V = IR

From b to c the voltage drop across R_2 is:

Voltages are different

why what does this mean?

$$iR_2 = \frac{1}{3}(5)$$
$$= 1.67\text{ V}$$

drop not voltage of electron

From c to d the voltage drop across R_3 is:

$$iR_3 = \frac{1}{3}(7)$$
$$= 2.33\text{ V}$$

$V_i - V_{drop} = V_f$ not given

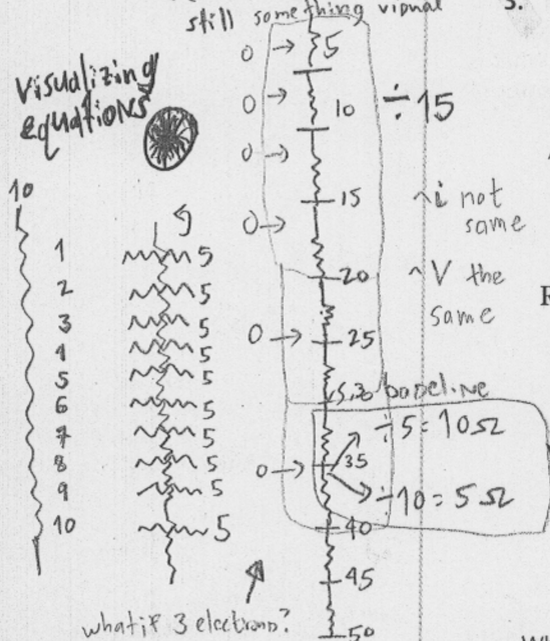
Designing half hour studies

make animation of this → even if not complete, is still something visual

try various numbers & find a pattern

Doug! (working w/ someone in the field)

visualizing equations



3. Resistors in Parallel

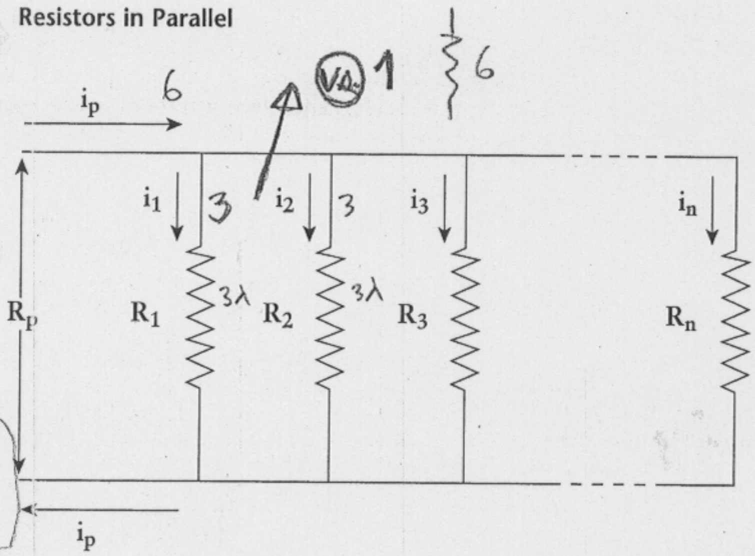


Figure 8.4

When resistors are wired in parallel, they are all wired with a common high potential terminal and a common low potential terminal (see figure 8.4). The effect of **adding resistors in parallel** is the same as that of increasing the cross-sectional area of a conductor. It increases the paths by which current can flow and thereby **decreases resistance** (there is also the analogy to viscous blood flow through capillaries—flow resistance is reduced when several capillaries are arranged in parallel). The rule for combining resistances in parallel is a bit more complicated than the previous rules. It states that the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances. In equation form this may be written as:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

Power $= i^2 V$

When resistors are in parallel, the voltage drop across each is the same and is equal to the voltage drop across the entire combination:

$$V_p = V_1 = V_2 = V_3 = \ldots V_n$$

Σ drop across all electrons in that set of current (hard b/c seems like over 1 electron)

Example: Consider two equal resistors wired in parallel. What is the equivalent resistance of the two?

Solution: The equation for summing resistors in parallel is:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Find the common denominator of the right-hand side and take the inverse to find:

5×10=50

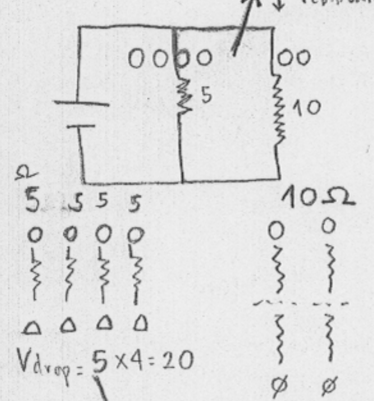
$$R_p = \frac{R_1 R_2}{(R_1 + R_2)}$$

Note:

Each additional resistor added in parallel acts to decrease the total resistance and thus increase the current.

Key concept

Q. given that ↑ electrons flow to ↓ resistance



is the voltage drop always over the entire voltage?

Note: Vdrop = 10×2 = 20

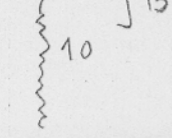
Parallel resistors experience the same voltage drop.

R. total = $\frac{10}{3}$ Ω (equation)

$6 \times \frac{10}{3} =$ 2×10 = 20 (Vdrop total)? over each resistors ↘ d.f. electrons go thru d.f. circuits

20 C/charge

total length if 1 resistor

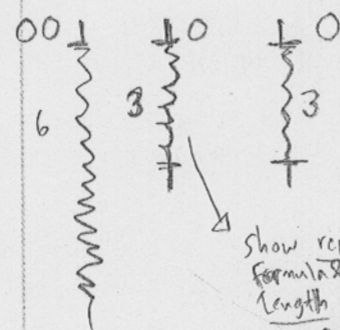


Since $R_1 = R_2$ in this special case, let $R = R_1 = R_2$:

resistance based ~ length of resistor

$$R_p = \frac{R^2}{2R} = \frac{R}{2}$$

In the above example, it is seen that the total resistance is **halved** by wiring two identical resistors in parallel. More generally, when n identical resistors are wired in parallel, the total resistance is given by R/n. Note that the voltage across each of the parallel resistors is equal, and that for equal resistances the current flowing through each of the resistors is also equal.



Show resistance formula & vary length

Δ resistor symbol

Example: Consider two resistors wired in parallel with $R_1 = 5\ \Omega$ and $R_2 = 10\ \Omega$. If the voltage across them is 10 V, what is the current through each of the two resistors?

Solution: First the current flowing through the whole circuit must be found. To do this, the combined resistance must be determined:

① need to compare to series, no sep. sieves

→ explain w/ flash generally, sieve

→ animation I tried to design....

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

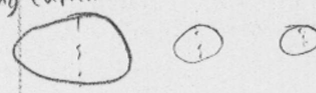
$$= \frac{1}{10} + \frac{1}{5}$$

$$= \frac{3}{10}$$

$$R_p = \frac{10}{3}\Omega \quad (3.3)$$

goes back to predetermining current principle from before

always less than the smallest resistor



Using Ohm's law to calculate the current flowing through the circuit gives:

Why???

amplification of an individual resistor

+ difficulty in keeping track of what is fixed & what is variable

?

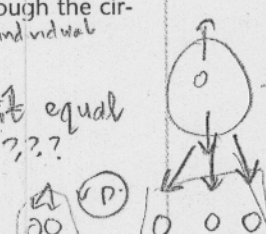
how to visualize

need equiv circuit w/ just one resistor....

$$i_p = \frac{V_p}{R_p} \longrightarrow \text{why does it equal this???}$$

$$= \frac{10}{\frac{10}{3}} \qquad 10 \to 20 = V \qquad \frac{10}{3}$$

$$= 3\text{ A} \qquad = 6\text{A}$$



size voltage sphere

(split)

number of electrons?

b/c when gets down to nothing = bottleneck of system

animate this →

Three Amps flow through the combination R_1 and R_2. Since the resistors are in parallel $V_p = V_1 = V_2 = 10$ V. Apply Ohm's law to each resistor individually:

click ampere, see electron visualization

visualized in the flash model

$$i_1 = \frac{V_p}{R_1} = \frac{10}{5} = 2\text{ A} \qquad R = \frac{20}{5} = 4\text{A}$$

$$i_2 = \frac{V_p}{R_2} = \frac{10}{10} = 1\text{ A} \qquad R = \frac{20}{10} = 2\text{A}$$

As a check, note that $i_p = 3\text{ A} = i_1 + i_2 = 2 + 1 = 3$ A. More current flows through the smaller resistance. In particular note that R_1 with half the resistance of R_2 has twice the current. Once i_p was found to be 3 A, the problem could have been solved by noting that because R_1 is half of R_2, $i_1 = 2i_2$, and $i_1 + i_2 = 3$ A.

D. CAPACITORS AND DIELECTRICS

1. Capacitors and Capacitance

Note:

The total charge on a capacitor is zero, +Q on one plate, and –Q on the other.

Note:

Capacitance is measured in Farads (F) and usually given in either $\mu F = 10^{-6}$ F, $nF = 10^{-9}$ F, or $pF = 10^{-12}$ F.

When two electrically neutral plates of metal are connected to a voltage source, positive charge builds up on the plate connected to the positive terminal, and an equal amount of negative charge builds up on the plate connected to the negative terminal. The two plate system stores charge and is called a **capacitor.** It is important to remember that charge collects on a capacitor any time there is a potential difference between the plates. The **capacitance** C of a capacitor is defined as the ratio of charge stored (meaning the absolute value of the charge on one plate) to the total potential difference across the capacitor. So, if a voltage difference V is applied across the plates of the capacitor and a charge Q collects on it (with +Q on the positive plate and –Q on the negative plate), then the capacitance is given by:

how much charge accumulates on d plate for every Volt
A, d, material effect this....

$$C = \frac{Q}{V}$$

+ + ↑↑
– –

how does this effect anything?

X The SI unit of capacitance is the **Farad** (where 1 F = 1 Coulomb/Volt). Because one Coulomb is such a large amount of charge, one Farad is a very large capacitance. Capacitances are therefore quoted in submultiples of the Farad such as microfarads (1 μF = 10^{-6} F), or nanofarads (1 nF = 10^{-9} F), or picofarads (1 pF = 10^{-12} F). Note also that the Farad should not be confused with the Faraday, the unit of charge equal to the charge on a mole of elementary charges (= 9.65×10^4 Coulombs).

The capacitance of a capacitor is dependent on the geometry of the two conducting surfaces. For the simple case of the parallel plate capacitor, the capacitance is given by:

$$C = \varepsilon_0 \frac{A}{d}$$

Note:

A charged parallel plate capacitor has a uniform electric field between the plates. The field direction is from the positive plate to the negative plate.

where ε_0 is the **permittivity of free space** ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m), A is the area of overlap of the two plates, and d is the separation of the two plates. The separation of charges sets up an electric field between the plates of the capacitor. The electric field between the plates of a parallel plate capacitor is a uniform field whose magnitude at any point is given by:

$$E = \frac{V}{d}$$

The direction of the electric field at any point between the plates is toward the negative plate and away from the positive plate.

2. Dielectric Materials

When an insulating material (such as glass, plastic, or certain metal oxides) is placed between the plates of a charged-up capacitor, the voltage across the capacitor decreases. Such insulating materials are called **dielectrics.** By lowering the voltage across the charged-up capacitor the dielectric has "made room for" even more charge, hence, the capacitance of the capacitor is increased. Dielectric materials are characterized by a dimensionless number called the **dielectric constant K,** which tells by what factor the capacitance of a capacitor is increased:

Note:

Dielectrics are insulators that increase the capacitance when inserted between the plates of a charged capacitor.

$$C' = KC$$

where C' is the new capacitance with the dielectric, and C is the original capacitance.

1. Example: The voltage across the terminals of an isolated 3 μF capacitor is 4 V. If a piece of ceramic having dielectric constant K = 2 is placed between the plates, find:

a. the new charge on the capacitor.
b. the new capacitance of the capacitor.
c. the new voltage across the capacitor.

Solution: a. The introduction of a dielectric by itself has no effect on the charge stored on the isolated capacitor. There is no new charge, so the charge is the same as before. The charge stored is therefore given by:

$$\begin{aligned} Q' &= Q \\ &= CV \\ &= (3 \times 10^{-6})(4) \\ &= 12 \times 10^{-6}\ C \\ &= 12\ \mu C \end{aligned}$$

b. By introducing a dielectric with a value of 2, the capacitance of the capacitor is doubled (C' = KC). Hence the new capacitance is 6 μF.

c. Using the relationship V' = Q'/C', the new voltage across the capacitor may be determined. Putting numbers into the equation gives:

$$V' = \frac{(12 \times 10^{-6})}{(6 \times 10^{-6})}$$

$$= 2\ V$$

2. Example: The voltage across the terminals of a 3 μF capacitor is 4 V. Now suppose a piece of ceramic having dielectric constant K = 2 is placed between the plates **and the voltage is held constant** (e.g., by a battery). What is the new charge on the capacitor?

Solution: By introducing the dielectric ceramic the capacitance of the capacitor has been altered. But because the voltage was held constant, the charge on the capacitor plates must have been altered. From the definition of dielectric constant and from the above example, it is clear that the new capacitance is:

dif from above

$$\begin{aligned} C' &= KC \\ &= 6\ \mu F \end{aligned}$$

But the new voltage is still 4 V, so the new charge must be:

$$\begin{aligned} Q' &= C'V' \\ &= (6 \times 10^{-6})(4) \\ &= 24 \times 10^{-6}\ \text{C} \\ &= 24\ \mu\text{C} \end{aligned}$$

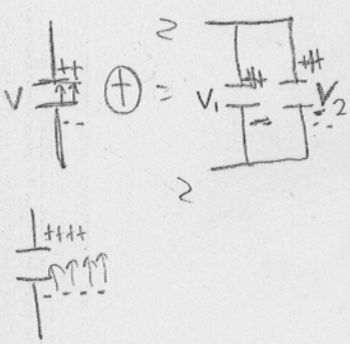
Since the original charge was $Q = CV = (3 \times 10^{-6})(4) = 12\ \mu C$, by keeping the voltage constant the battery had to supply an additional $+12\ \mu C$ of charge to the positive plate and $-12\ \mu C$ to the negative plate.

Note:

Each capacitor added in parallel acts to increase the total capacitance of the combination.

Note:

As with resistors, the voltage across each parallel capacitor is the same.



Note:

Each capacitor added in series acts to decrease the total capacitance of the combination.

3. Capacitors in Parallel

When wired in parallel, capacitors can be added directly. The capacitors wired in parallel can be thought of as combining to form a single capacitor with increased capacitance. Since the wire from one capacitor to the next is a conductor and an equipotential surface, the potential of all the plates on one side are the same (see figure 8.5).

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n$$

(increasing area of overlap)

The voltage across each parallel capacitor is the same, and is equal to the voltage across the entire combination:

V = QC

$$V_p = V_1 = V_2 = V_3 = \ldots = V_n$$

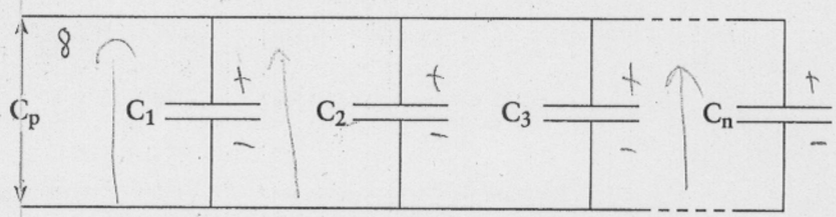


Figure 8.5

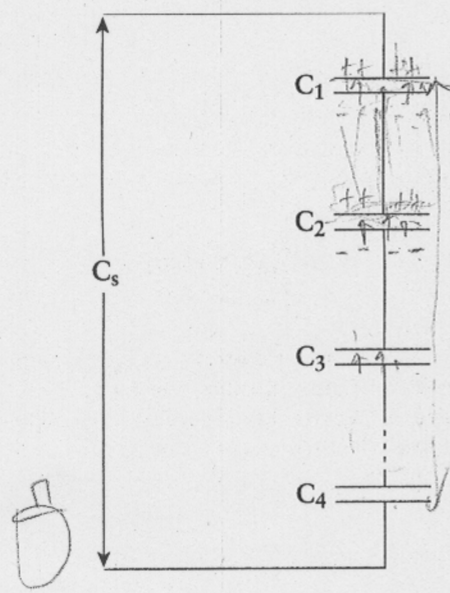
4. Capacitors in Series

Each additional capacitor added in series decreases the total capacitance of the circuit, so just as for resistors in parallel, the reciprocal of the total capacitance in series is equal to the sum of the reciprocals of the individual capacitances (see figure 8.6).

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

For capacitors in series, the total voltage is the sum of the individual voltages:

$$V_s = V_1 + V_2 + V_3 + \ldots + V_n$$



Cancels

extending area of dielectric

no power thru both ends

Figure 8.6

Note:

As with resistors, the total voltage drop across a series of capacitors is the sum of the voltage drops across each separately.

E. A SUMMARY OF CIRCUIT ELEMENT ADDITION

SERIES

$$R_s = R_1 + R_2 + R_3 + \ldots + R_n$$

i same
V d.b.

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

V d.b = sum individual

PARALLEL

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

i d.b
V same

$$C_p = C_1 + C_2 + C_3 + \ldots + C_n$$

V same

ALTERNATING CURRENT

A. ALTERNATING CURRENT

Alternating current (AC) changes its direction of flow periodically. The most common form of AC current oscillates in a sinusoidal way as shown in figure 8.7. Note that for half of the cycle the current flows in one direction, and for the other half of the cycle the current flows in the opposite direction. Such a current can be described by the equation

$$i = I_{max} \sin(2\pi f t)$$
$$= I_{max} \sin(\omega t)$$

where i is the instantaneous current at the time t, I_{max} is the maximum current, f is the frequency, and $\omega = 2\pi f$ is the angular frequency.

The most common sinusoidal current is the ordinary AC house current that oscillates with a frequency f of 60 Hz. In some countries, such as England, the frequency is 50 Hz.

Note:

Negative current means that the current flows in the opposite direction to the direction it has when it's positive.

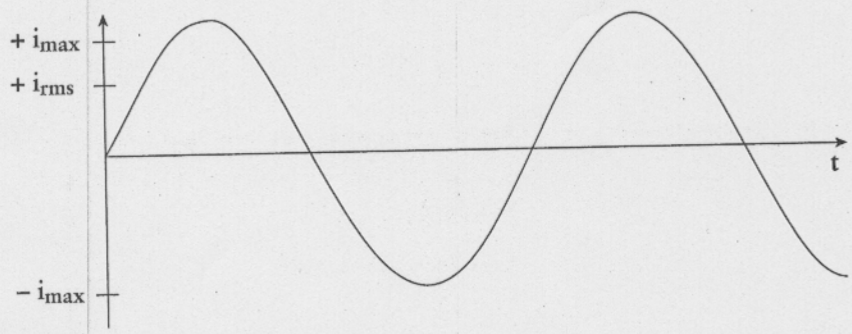


Figure 8.7

B. RMS CURRENT

In alternating current circuits the magnitude of the current varies from a maximum positive value to a minimum negative value. A problem arises when one tries to calculate the average current for sinusoidal AC currents: for one cycle, the sum of the positive current flowing in one direction is exactly canceled by the sum of the negative current that flows in the other direction. Yet there is AC current; it delivers power. Consider the power dissipated in a resistor R that carries an AC current i. It is given by the equation $P = i^2R$. Therefore, in order to find the average power dissipated, we must find the average of i^2 over one period. This is equal to I_{rms}^2, where I_{rms} is the root-mean-square (rms) current given by:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

Note:

The average value of an AC current is zero since half the time it's positive and the other half of the time it's negative by the same amount.

Example: What is the rms current of an AC signal that will produce a maximum current of 1.00 A?

Solution:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1.00}{\sqrt{2}} = \frac{1.00}{1.41} = 0.71 \text{ A}$$

C. RMS VOLTAGE

Voltage in AC circuits, like current, is sinusoidal and changes sign back and forth over time. It can be described by an equation similar to the equation for sinusoidal current. So just as for current, one can calculate an **rms voltage:**

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

Example: The AC current used in a home is frequently called "120 V AC." Assuming that this refers to the rms voltage, what is the maximum voltage?

Solution: Using the above equation gives:

$$V_{max} = \sqrt{2}V_{rms} = \sqrt{2}(120) = 170 \text{ V}$$

Note:

Average power in an AC circuit is not zero since $P = i^2R$ and i^2 is always positive.

Note:

Voltage in AC circuits alternates between positive and negative values. A negative value means that what was the positive terminal is now the negative and what was the negative terminal is now the positive.